بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals
DEPARTMENT OF CIVIL ENGINEERING
Second Semester 1433-34 / 2012-13 (122)

CE 203 STRUCTURAL MECHANICS I

Major Exam I

KEY SOLUTION

Problem	GRADER
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Problem 1: (20 points)

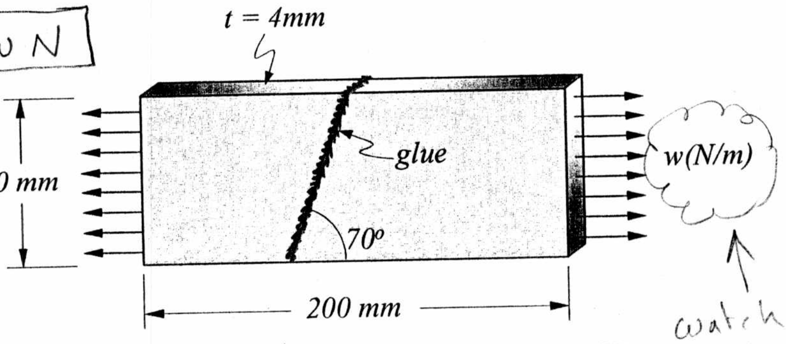
The given thin plate is made of two parts glued together as shown. The plate is subjected to an axial distributed load w *(N/m)*. Determine the largest value of w that can be applied.

For the plate material : ultimate normal stress = 60 *MPa*

For the glue : ultimate normal stress = 30 *MPa* , and ultimate shear stress = 15 *MPa*

For the whole problem, use safety factor *S.F.* = 3

Applied force $\boxed{F = .08w\ N}$



watch units

a) check plate :

$$\sigma_{all} = \frac{60}{3} = 20\,MPa$$

$$\sigma_{all} = 20 = \frac{F}{A} = \frac{.08w}{(80)(4)} \rightarrow \boxed{w = 80{,}000\ N/m} \quad (1)$$

b) check glue

$N = (.08w)\sin 70$

$V = (.08w)\cos 70$

To find the inclined surface area

$$d = \frac{80}{\sin 70}$$

• check glue normal : $\sigma_{all} = \frac{30}{3} = 10\,MPa = \frac{\text{force}}{\text{area}} = \frac{.08w\sin 70}{(4)(80/\sin 70)}$

$$\boxed{w = 45{,}299\ N/m} \quad (2)$$

• check glue shear : $\tau_{all} = \frac{15}{3} = 5\,MPa = \frac{\text{force}}{\text{area}} = \frac{.08w\cos 70}{(4)(80/\sin 70)}$

$$\boxed{w = 62{,}229\ N/m} \quad (3)$$

$$\boxed{w_{max} = 45{,}299\ N/m}$$ Answer.

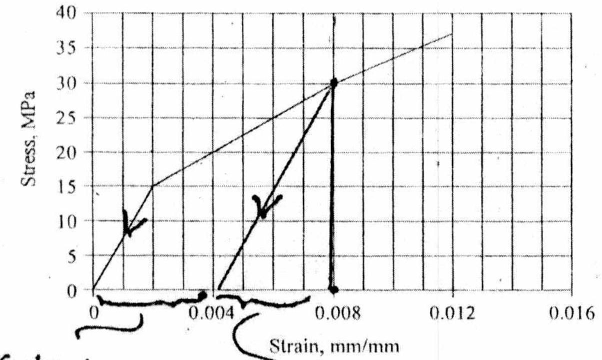
Compare (1), (2), (3)
the smallest controls

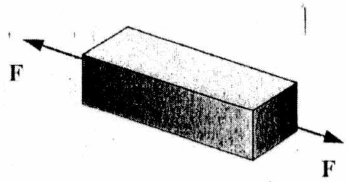
Problem 2: (20 points)

A bar with the stress-strain diagram shown was originally 1 m long with a square cross-sectional area of 100 mm x 100 mm.
When an axial tension load F is applied, the square cross-section became 99.95 mm x 99.95 mm.
Determine the following:

6 a) The magnitude of the applied force F.
3 b) The final length of the bar when the load F is applied.
2 c) The final length of the bar when the load F is released.
5 d) The final length of the bar when the applied load is 300 kN.
4 e) The final length of the bar when the 300 kN load is released.

Poisson's ratio, ν = 0.25





Solution

(a) $\nu = -\dfrac{\varepsilon_{lat}}{\varepsilon_{long}} \Rightarrow \varepsilon_{long} = \dfrac{-\varepsilon_{lat}}{\nu}$

$\varepsilon_{lat} = \dfrac{99.95 - 100}{100} = -0.0005 \ \frac{mm}{mm}$ 2

$\varepsilon_{long} = \dfrac{-(-0.0005)}{0.25} = 0.002 \ \frac{mm}{mm}$ 1

From σ-ε diagram when $\varepsilon_{long} = 0.002 \Rightarrow \boxed{\sigma = 15 \text{ MPa}}$ 2

$\sigma = \dfrac{P}{A_o} \Rightarrow P = \sigma A_o = 15 \times 10000 = \boxed{150000 \text{ N} = 150 \text{ kN}}$

(b) $\varepsilon_{long} = \dfrac{L_f - L_o}{L_o} \Rightarrow L_f = (\varepsilon_{long} \times l_o) + l_o = \boxed{1.002 \text{ m} = l_f}$ 3

(c) when the load F is released will go back or original Length $\sigma = \sigma_y$,
$\therefore L_f = 1$ m. 2

(d) $\sigma = \dfrac{300000}{10000} = 30$ MPa, in the plastic range. 2
at $\sigma = 30$ MPa, $\varepsilon_{long} = 0.008$ mm/mm 2
$L_f = (0.008)(1) + 1 = \boxed{1.008 \text{ m}}$ 1

(e) $E = \dfrac{\sigma}{\varepsilon_{long}} = \dfrac{15}{0.002} = 7500$ MPa 3

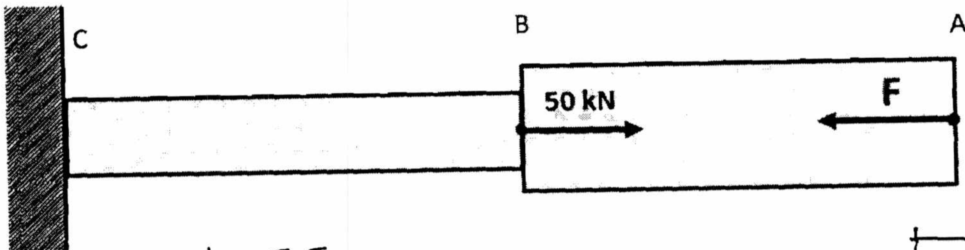
recovered strain $= \dfrac{30}{7500} = 0.004 \ \frac{mm}{mm}$, or directly form the graph
permentant strain $= 0.008 - 0.004 = .004$ mm/mm
$L_f = (1 \times .004) + 1 = \boxed{1.004 \text{ m}}$ 1

Problem 3: (20 points)

The rods AB and BC are subjected to the *loads and temperature changes* shown in the figure and table below. Determine the **maximum allowable force F** that can be applied (in the shown direction) if

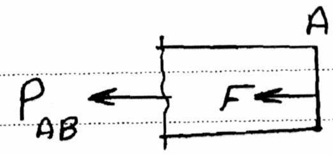
- the maximum allowable normal stress in AB is 150 MPa (tension or compression), and
- the maximum allowable normal stress in BC is 100 MPa (tension or compression), and
- the maximum allowable displacement of point A is $5\,(10)^{-4}$ m.

Member \ *Properties*	L (m)	A (m^2)	E (GPa)	ΔT (°C)	α (/°C)
AB	0.5	$4\,(10)^{-4}$	200	+40	$20\,(10)^{-6}$
BC	0.6	$3\,(10)^{-4}$	100	-60	$15\,(10)^{-6}$



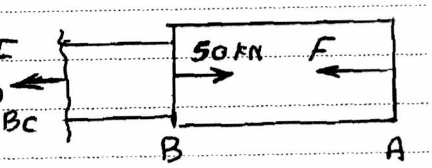
(PB)

(1) FBD : AB $\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$



(1) $-F - P_{AB} = 0 \Rightarrow P_{AB} = -F$ "C"

(1) FBD : BC $\xrightarrow{+} \Sigma F_x = 0 \Rightarrow$



(1) $-F + 50(10)^3 - P_{BC} = 0 \Rightarrow P_{BC} = 50(10)^3 - F$

$\sigma^{AB}_{max\ allow} = P_{AB}/A_{AB} = \pm 150\,(10)^6 \Rightarrow$

(2) $-F/4(10)^{-4} = -150\,(10)^6 \Rightarrow F^{①}_{max} = 60$ kN

$\sigma^{BC}_{max\ allow} = P_{BC}/A_{BC} = \pm 100\,(10)^6 \Rightarrow$

(2) $[50\,(10)^3 - F]/3\,(10)^{-4} = -100\,(10)^6 \Rightarrow F^{②}_{max} = 80$ kN

displ. of A $= \Sigma\delta = (\delta_{mech} + \delta_{therm})_{AB} + (\delta_{mech} + \delta_{therm})_{BC}$

(2) $\delta^{AB}_{mech} = e_{load} = \dfrac{PL}{AE} = \dfrac{-F\,(0.5)}{4\,(10)^{-4}\,200\,(10)^9} = -6.25\,(10)^{-9}\,F$ «←»

(1) $\delta^{AB}_{therm} = e_{\Delta T} = \alpha\,\Delta T L = 20\,(10)^{-6}\,(+40)\,(0.5) = +4\,(10)^{-4}$ m «→»

(2) $\delta^{BC}_{mech} = e_{load} = \dfrac{PL}{AE} = \dfrac{[50(10)^3 - F]\,(0.6)}{3(10)^{-4}\,100\,(10)^9} = 1\,(10)^{-3} - 2\,(10)^{-8}\,F$

(2) $\delta^{BC}_{therm} = e_{\Delta T} = \alpha\,\Delta T L = 15\,(10)^{-6}\,(-60)\,(0.6) = -5.4\,(10)^{-4}_m$ «←»

displ. of A $= -6.25\,(10)^{-9}\,F + 4\,(10)^{-4} + 1\,(10)^{-3} - 2\,(10)^{-8}\,F - 5.4\,(10)^{-4}$

(1) $= 8.6\,(10)^{-4} - 2.625\,(10)^{-8}\,F \Rightarrow$

$8.6\,(10)^{-4} - 2.625\,(10)^{-8}\,F = -5\,(10)^{-4}$ [Note the minus sign! Why ?!]

(2) $\Rightarrow F^{③}_{max} = 1.36\,(10)^{-3}/2.625\,(10)^{-8}$

$= 51.81$ kN

(2) $F_{max} = \min(F^{①}_{max}, F^{②}_{max}, F^{③}_{max}) \Rightarrow \boxed{F_{max} = 51.81 \text{ kN}}$

«Why ?!» {Note that σ_{BC} is still "ok" !}

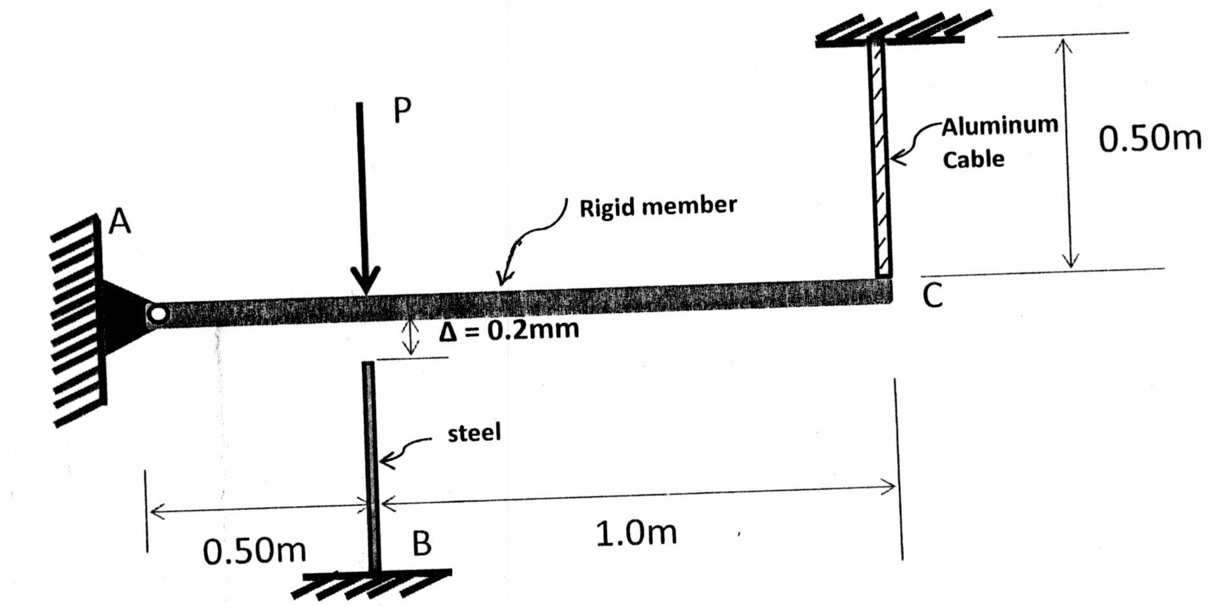
Problem 4: (20 points)

Rigid member AC is hinged at A and is supported by an aluminum cable at C. Before applying the load, AC was horizontal and a gap, $\Delta = 0.2$ mm separated it from a steel rod as shown.

If P = 24 kN, determine the following:

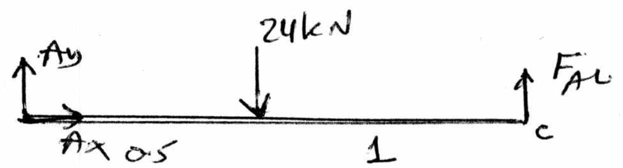
a) the stress in the aluminum cable.
b) the displacement of point C.

$E_{aluminum} = 70$ **GPa,** $E_{steel} = 200$ **GPa,** $L_{steel} = 0.5$ **m**
$A_{aluminum} = A_{steel} = 50$ **mm^2**



(I) First need to check if the gap closes or not and the problem need to be treated as statically determinate :

FBD :



(1) $\sum M_{@A} = 0 +\circlearrowleft \quad 24\times10^3(0.5) = 1.5\,(F_A) \Rightarrow F_A = 8000N$

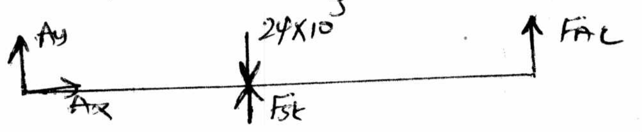
(1) $\delta_c = \frac{F_{AL}\, l}{E_{AL} A_{AL}} = \frac{(8000)(0.5)}{(70\times10)^9 (50\times10^{-6})} = 1.142$ mm

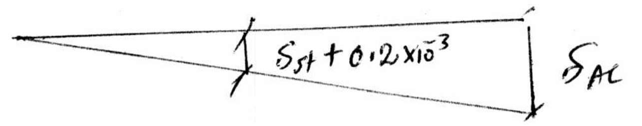
(1) $\delta_B = \delta_c\left(\frac{.5}{1.5}\right) = 0.38$ mm > 0.2 mm

∴ The problem is statically indeterminate as the gap c

(II) Now treat the problem as statically indeterminat prob

(i) FBD





(δ - in m)

Equilibrium Eq:

(5) $\sum M_{@A} = 0 + \uparrow \quad 1.5 F_{AL} + F_{st}(0.5) = 24 \times 10^3 (0.5)$ — (A)

Compatability Equation

(6) $\dfrac{\delta_{st} + 0.2\times10^{-3}}{0.5} = \dfrac{\delta_{AL}}{1.5} \Rightarrow 3\delta_{st} + 0.6\times10^{-3} = \delta_{AL}$

$$3 \frac{F_{st}(0.5)}{(200\times10^9)(50\times10^{-6})} + 0.6\times10^{-3} = \frac{F_{AL}(0.5)}{(70\times10^9)(50\times}$$

$$1.5 F_{st} + .6\times10^4 = 1.429 F_{AL} \quad -$$

solving (A) & B $\Rightarrow$

(1) $F_{st} = 2747$ N

$F_{AL} = 7084$ N

(2) (a) stress in Aluminium $= \dfrac{F_{AL}}{A_{AL}} = \dfrac{7084}{50\times10^{\circ}} = 142$ MPa

(2) (b) Displacement of point c $= \dfrac{F_{AL} l_{AL}}{E_{AL} A_{AL}} = \dfrac{(7084)(0.5)}{(70\times10^9)(50\times10^{-6})}$

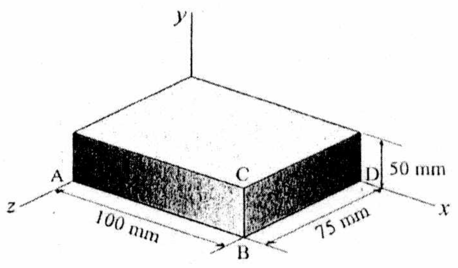
$\delta_c = 1.012 \times 10^{-3}$ m

$\delta_c = 1.012$ mm

Problem 5: (20 points)

The steel block shown is subjected to a uniform pressure p on all the faces. Knowing that the change in length of edge AB is -30×10^{-3} mm and using E = 200 GPa, and G = 75 GPa, determine the followings:

8 a) The magnitude of the applied pressure, p.
3 b) The strains in the x, y, and z directions.
6 c) The new length of AB, CB, and BD after the application of the uniform pressure p.
3 d) The change in volume, using any approach.



Initial Dimensions

solution

(a)

$$\varepsilon_x = \frac{(\Delta L)_{AB}}{L_{AB}} = \frac{-30\times10^{-3}}{100} = -3\times10^{-4}\ mm/mm \quad (2)$$

$$\varepsilon_x = -3\times10^{-4} = \frac{1}{200\times10^{9}}\left[-P - 0.333(-P-P)\right]$$

$$\boxed{P = 179.64\ MPa} \quad (4)\ \text{Compression}$$

(b)

$$\boxed{\varepsilon_x = -3\times10^{-4}} \quad (1), \qquad G = \frac{E}{2(1+\nu)}, \quad 75\times10^{9} = \frac{200\times10^{9}}{2(1+\nu)} \Rightarrow \nu = 0.333 \quad (2)$$

$$\varepsilon_y = \frac{1}{200\times10^{9}}\left[-179.64\times10^{6} - 0.333(-2 * 179.64\times10^{6})\right]$$

$$\boxed{\varepsilon_y = -3\times10^{-4}\ mm/mm} \quad (1)$$

similarly $\rightarrow$ $\boxed{\varepsilon_z = -3\times10^{-4}\ mm/mm}$ (1)

(c)

$$(L_{AB})_{new} = (-30\times10^{-3}) + 100 = \boxed{99.97\ mm} \quad (2)$$

$$(L_{CB})_{new} = (50 * -3\times10^{-4}) + 50 = \boxed{49.985\ mm} \quad (2)$$

$$(L_{BD})_{new} = (75 * -3\times10^{-4}) + 75 = \boxed{74.9775\ mm} \quad (2)$$

(d) change in Volume = ΔV =

$$(99.97)(49.985)(74.9775) - (100)(50)(75) =$$

$$\boxed{\Delta V = -337.40\ mm^3}$$

$-337.4\ mm^3$ (3)

or

$$e = \frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\frac{\Delta V}{375000} = 3(-3\times10^{-4})$$

$$\Delta V = -337.5\ mm^3$$