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**Problem 1:** (20 points)

The given thin plate is made of two parts glued together as shown. The plate is subjected to an axial distributed load \( w \) (N/m). Determine the largest value of \( w \) that can be applied.

For the plate material: ultimate normal stress = 60 MPa

For the glue: ultimate normal stress = 30 MPa, and ultimate shear stress = 15 MPa

For the whole problem, use safety factor \( S.F. = 3 \)

**Applied force**

\[ F = 0.08w \text{ N} \]

\[ t = 4 \text{ mm} \]

\[ w(\text{N/m}) \]

\[ w = \frac{80000}{80} \text{ N/m} \]

\[ \sigma_{\text{all}} = \frac{60}{3} = 20 \text{ MPa} \]

\[ \sigma_{\text{all}} = \frac{F}{A} = \frac{0.08w}{(80)(4)} \]

\[ \omega = 45299 \text{ N/m} \]

\[ \text{To find the inclined surface area} \]

\[ d = \frac{80}{\sin 70} \]

\[ \text{Check glue normal:} \]

\[ \sigma_{\text{all}} = \frac{30}{3} = 10 \text{ MPa} = \frac{0.08w \sin 70}{(4)(80/\sin 70)} \]

\[ \omega = 45299 \text{ N/m} \]

\[ \text{Check glue shear:} \]

\[ \tau_{\text{all}} = \frac{15}{3} = 5 \text{ MPa} = \frac{0.08w \cos 70}{(4)(80/\sin 70)} \]

\[ \omega = 62229 \text{ N/m} \]

\[ \omega_{\text{max}} = 45299 \text{ N/m} \]

Compare 0, 2, 3

**Answer.**
Problem 2: (20 points)

A bar with the stress-strain diagram shown was originally 1 m long with a square cross-sectional area of 100 mm x 100 mm. When an axial tension load F is applied, the square cross-section became 99.95 mm x 99.95 mm. Determine the following:

a) The magnitude of the applied force F.
b) The final length of the bar when the load F is applied.
c) The final length of the bar when the load F is released.
d) The final length of the bar when the applied load is 300 kN.
e) The final length of the bar when the 300 kN load is released.

Poisson's ratio, \( v = 0.25 \)

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Solution

\[ E_{\text{lat}} = \frac{99.95 - 100}{100} = -0.0005 \text{ mm/mm} \]

\[ E_{\text{long}} = \frac{-(-0.0005)}{0.25} = 0.002 \text{ mm/mm} \]

From \( \sigma - \varepsilon \) diagram when \( E_{\text{long}} = 0.002 \Rightarrow \sigma = 15 \text{ MPa} \)

\[ E_{\text{lat}} = \frac{L_p - L_0}{L_0} \Rightarrow L_p = (E_{\text{lat}} x L_0) + L_0 = (1.002 \text{ m} \times 1\) m = 1 m

\[ \sigma = \frac{300000}{10000} = 30 \text{ MPa} \] in the plastic range.

\[ E_{\text{long}} = 0.008 \text{ mm/mm} \]

\[ L_p = (0.008)(1) + 1 = 1.008 \text{ m} \]

\[ E = \frac{E_{\text{lat}}}{E_{\text{long}}} = \frac{15}{0.002} = 7500 \text{ MPa} \]

Recover Strain \( = \frac{30}{7500} = 0.004 \text{ mm/mm} \) or directly form graph

\[ L_p = (1 \times 0.004) + 1 = 1.008 \text{ m} \]
**Problem 3:** (20 points)

The rods AB and BC are subjected to the loads and temperature changes shown in the figure and table below. Determine the maximum allowable force \( F \) that can be applied (in the shown direction) if

- the maximum allowable normal stress in AB is 150 MPa (tension or compression), and
- the maximum allowable normal stress in BC is 100 MPa (tension or compression), and
- the maximum allowable displacement of point A is 5 (10)^{-3} m.

<table>
<thead>
<tr>
<th>Properties</th>
<th>L (m)</th>
<th>A (mm^2)</th>
<th>E (GPa)</th>
<th>ΔT (°C)</th>
<th>α (1/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0.5</td>
<td>4 (10)^{-3}</td>
<td>200</td>
<td>-40</td>
<td>20 (10)^{-6}</td>
</tr>
<tr>
<td>BC</td>
<td>0.6</td>
<td>3 (10)^{-4}</td>
<td>100</td>
<td>-60</td>
<td>15 (10)^{-6}</td>
</tr>
</tbody>
</table>

1. **FBD for AB:** \( F \rightarrow \Delta F_x = 0 \Rightarrow P_{AB} = -F \)  
2. **FBD for BC:** \( -F + 50 (10)^{3} - \sigma_{BC} = 0 \Rightarrow \sigma_{BC} = 50 (10)^{3} - F \)
3. \[ \sigma_{allow, AB} = \frac{P_{AB}}{A_{AB}} \leq 150 (10)^{3} \Rightarrow P_{BC} = 50 (10)^{3} - F \]
4. \[ \sigma_{allow, BC} = \frac{P_{BC}}{A_{BC}} \leq 100 (10)^{3} \Rightarrow F_{max} = 60 \text{ kN} \]
5. \[ \frac{50 (10)^{3} - F}{3 (10)^{4}} = -100 (10)^{3} \Rightarrow F_{max} = 80 \text{ kN} \]
6. Displ. of A: \( \Sigma \Delta = (\sigma_{mech} + \sigma_{therm})_{AB} + (\sigma_{mech} + \sigma_{therm})_{BC} \)
7. \[ \sigma_{mech} = \frac{P_{AB}}{A_{AB}} = \frac{F (0.5)}{4 (10)^{-3}} = -6.25 (10)^{-3} F \]
8. \[ \sigma_{therm} = A_{BT} \Delta TL = 2 \times (10)^{-9} (40) (0.5) = 4 (10)^{-9} \text{ m} \]
9. \[ \sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{50 (10)^{3} - F}{3 (10)^{4}} = 1 (10)^{-3} - 2 (10)^{-8} F \]
10. \[ \sigma_{therm} = A_{BT} \Delta TL = 15 (10)^{-9} (-60) (0.6) = -5.4 (10)^{-9} \]
11. \[ \text{Displ. of A:} \quad \Delta A = -6.25 (10)^{-3} F + 4 (10)^{-9} + 1 (10)^{-3} - 2 (10)^{-8} F = -5.4 (10)^{-9} \]
12. \[ F_{max} = \frac{1.36 (10)^{-3}}{2.625 (10)^{-8}} \Rightarrow F_{max} = 51.81 \text{ kN} \]

The maximum allowable force \( F_{max} \) is 51.81 kN.
Problem 4: (20 points)

Rigid member AC is hinged at A and is supported by an aluminum cable at C. Before applying the load, AC was horizontal and a gap, $\Delta = 0.2$ mm separated it from a steel rod as shown.

If $P = 24$ kN, determine the following:

a) the stress in the aluminum cable.

b) the displacement of point C.

$E_{\text{aluminum}} = 70$ GPa, $E_{\text{steel}} = 200$ GPa, $L_{\text{steel}} = 0.5$ m

$A_{\text{aluminum}} = A_{\text{steel}} = 50$ mm$^2$
Now treat the problem as statically indeterminate.

**Equilibrium Eqn:**
\[ \sum \mathbf{M}_A = 0 \]
\[ 1.5 \mathbf{F}_{Ac} + \mathbf{F}_{st} (0.5) = 24 \times 10^3 \ (0.5) \]

**Compatibility Eqn:**
\[ \frac{8st + 0.2 \times 10^3}{0.5} = \frac{\mathbf{S}_{AC}}{1.5} \Rightarrow 3 \mathbf{S}_{st} + 0.6 \times 10^{-3} = \mathbf{S}_{AC} \]

\[ \frac{\mathbf{F}_{st} (0.5)}{(2\times10^9)(50 \times 10^{-6})} + 0.6 \times 10^{-3} = \frac{\mathbf{F}_{Ac} (0.5)}{(2\times10^9)(50 \times 10^{-6})} \]

\[ 1.5 \mathbf{F}_{st} + 0.6 \times 10^{-3} = 1.4 \times 10^{-3} \mathbf{F}_{Ac} \]

**Solving:**

\[ \mathbf{F}_{st} = 2747 \ N \]
\[ \mathbf{F}_{Ac} = 7084 \ N \]

**Stress in Aluminum:**
\[ \sigma = \frac{\mathbf{F}_{Ac}}{A_{Al}} = \frac{7084}{5 \times 10^{-4}} = 142 \ MPa \]

**Displacement at Point C:**
\[ \delta_C = \frac{\mathbf{F}_{Ac} l_{AC}}{E_{Al} A_{Al}} = \frac{(7084)(0.5)}{(2 \times 10^9)(50 \times 10^{-6})} \]

\[ \delta_C = 1.012 \times 10^{-3} \ m \]

\[ \delta_C = 1.012 \ mm \]
Problem 5: (20 points)

The steel block shown is subjected to a uniform pressure \( p \) on all the faces. Knowing that the change in length of edge AB is \(-30 \times 10^{-3}\) mm and using \( E = 200 \text{ GPa} \), and \( G = 75 \text{ GPa} \), determine the followings:

a) The magnitude of the applied pressure, \( p \).

b) The strains in the x, y, and z directions.

c) The new length of AB, CB, and BD after the application of the uniform pressure \( p \).

d) The change in volume, using any approach.

Solution

\[ E_x = \frac{(\Delta L)_{AB}}{L_{AB}} = -3 \times 10^{-3} \text{ mm/mm} \]

\[ E_x = -3 \times 10^{-4} = \frac{1}{200 \times 10^9} \left[ -p - 0.333 (-p - p) \right] \]

\[ P = 179.64 \text{ MPa} \]

\[ E_y = \frac{1}{200 \times 10^9} \left[ -179.64 \times 10^{-9} - 0.333 (-2 \times 179.64 \times 10^{-9}) \right] \]

\[ E_y = -3 \times 10^{-7} \text{ mm/mm} \]

\[ (L_{AB})_{new} = (-30 \times 10^{-3}) + 100 = 99.7 \text{ mm} \]

\[ (L_{CB})_{new} = (50 \times -3 \times 10^{-4}) + 50 = 49.985 \text{ mm} \]

\[ (L_{BD})_{new} = (75 \times -3 \times 10^{-4}) + 75 = 74.975 \text{ mm} \]

\[ \text{change in volume } = \Delta V = (99.7)(49.985)(74.975) - (100)(50)(75) = -337.41 \text{ mm}^3 \]

\[ \frac{\Delta V}{337.41} = \frac{\epsilon}{\epsilon_x + \epsilon_y + \epsilon_z} \]

\[ \frac{\Delta V}{337.41} = 3 (-3 \times 10^{-4}) \]

\[ \Delta V = -337.75 \text{ mm}^3 \]